Recall:

Definition 0.1. A function $f : A \to B$ is an ***injection*** if, for any two distinct elements of A, say $a \neq a'$, we have $f(a) \neq f(a')$. A function is a ***surjection*** if, for ever $b \in B$, there exists an element $a \in A$ such that $f(a) = b$. A function that is both an injection and a surjection is called a ***bijection***.

Setup

We'll think about equivalence classes of functions $f : N \to K$, where N and K are finite sets. By convention, we'll let $n = |N|$ and $k = |K|$. In this situation, we consider functions f and g equivalent (so they are in the same equivalence class) if $f(x) = g(x)$ for each $x \in N$.

The standard way to think of this setup is to think about placing n **balls** into k **boxes**. You're welcome to use this setup, or you can define N and K to be finite sets that make more sense to you, as long as you can think about the elements of N being put into elements of K.

Decide within your table what you want to use as N and K.

N is...

K is...

Exploring types of functions

We're going to divide and conquer a bit to see how many equivalence classes of functions there are for different pairs of numbers (n, k) We can also think of this as asking

"How many ways are there to place the elements of N into elements of K for different sizes of N and K?"

As it turns out, the answer to this question ***depends on whether we can tell the elements of*** N ***and/or the elements of*** K ***apart.*** It also depends on what kind of functions we're thinking of.

For our table, we can / can not tell the elements of N apart and we can / can not tell the elements of K apart. That is...

Today, you're going to spend some time getting an answer to the question above by considering different restrictions on functions for your setup, and we'll bring our answers together at the end of class time.

As you fill in the tables on the following sheets, **start with very small** n and k (maybe 0 to 3) and fill in the number of functions in the corresponding box. Once you've worked through several pairs, **try to identify a pattern**, then see if your pattern works for larger n and k.

No restriction on functions

Room to show work:

$n\backslash k$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Surjections only

Room to show work:

$n\backslash k$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Injections only

Room to show work:

$n \backslash k$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Bijections only

Room to show work:

$n \backslash k$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Exploring distinguishability

Instead of fixing our perspective of whether or not we can tell the elements of N and K apart, we might decide to fix what type of functions we want to focus on.

For this exploration we are only considering surjections / injections / no restrictions .

We're going to use slightly more mathematical language as we consider whether we can tell the elements of N and/or the elements of K apart. We say the elements of N are ***distinguishable*** if we can tell them apart, and we say they are ***indistinguishable*** if we can't tell them apart.

Elements of both N and K indistinguishable

Room to show work:

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Elements of N indistinguishable, elements of K distinguishable

Room to show work:

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Elements of N distinguishable, elements of K indistinguishable

Room to show work:

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Elements of both N and K distinguishable

Room to show work:

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						